SET 2016

PAPER – II

MATHEMATICAL SCIENCES

Signature of the Invigilator	Question Booklet No. 040911
1.	OMR Sheet No.. ....
Subject Code 04	**ROLL No.**
Time Allowed : 75 Minutes	**Max. Marks : 100**
No. of pages in this Booklet : **12**	No. of Questions : **50**

INSTRUCTIONS FOR CANDIDATES

1. Write your Roll No. and the OMR Sheet No. in the spaces provided on top of this page.
2. Fill in the necessary information in the spaces provided on the OMR response sheet.
3. This booklet consists of fifty (50) compulsory questions each carrying 2 marks.
4. Examine the question booklet carefully and tally the number of pages/questions in the booklet with the information printed above. **Do not accept a damaged or open booklet**. Damaged or faulty booklet may be got replaced within the first **5** minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time given.
5. Each Question has four alternative responses marked (A), (B), (C) and (D) in the OMR sheet. You have to completely darken the circle indicating the most appropriate response against each item as in the illustration.

   (A) (B) ● (D)

6. All entries in the common OMR response sheet for Papers I and II are to be recorded in the original copy only.
7. Use only Blue/Black Ball point pen.
8. Rough Work is to be done on the blank pages provided at the end of this booklet.
9. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Sheet, except in the spaces allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, you will render yourself liable to disqualification.
10. You have to return the Original OMR Sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. **You are, however, allowed to carry the test booklet and the duplicate copy of OMR Sheet** on conclusion of examination.
11. Use of any calculator, mobile phone or log table etc. is strictly prohibited.
12. **There is no negative marking.**

04-16

SEAL

PAPER-II

MATHEMATICAL SCIENCES

1. Which of the following statements gives completeness axiom ?
   (A) Every bounded infinite subset of R has a limit point in R
   (B) Every subset of R which is bounded above has supremum
   (C) Every closed and bounded subset of R is compact
   (D) Every K-cell is compact

2. If $s_n = (-1)^n \left(1 + \frac{1}{n}\right)$ for all $n \in N$ is a sequence of real numbers. Then $\lim_{sup}$ and $\lim_{inf}$ of $\{s_n\}$ are :
   (A) 1, 0 (B) 0, –1
   (C) 1, – 1 (D) 1, 1

3. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is a :
   (A) Convergent series
   (B) Divergent series
   (C) Conditionally convergent series
   (D) None

4. Suppose $a, b \in R$ with $a < b$. Then $[a, b)$ is a :
   (A) Countable set
   (B) Almost countable set
   (C) Uncountable set
   (D) Compact set in R

5. Suppose $A = \left\{1, \frac{1}{2}, \frac{1}{4}, \ldots \frac{1}{2n} \ldots\right\}$ and $B = \left\{\frac{1}{3}, \frac{1}{5}, \ldots \frac{1}{2n-1}, \ldots\right\}$ are two subsets of the metric space R with respect to usual metric d then $d(A, B) =$
   (A) 1/2
   (B) 1
   (C) 1/3
   (D) 0

6. Suppose $f(x) = \frac{\sin x}{x}$ if $x \neq 0$ and $f(x) = 1$ if $x = 0$. Then f is :
   (A) continuous at 0
   (B) discontinuous at 0
   (C) has removable discontinuity at 0
   (D) (B) but not (C)

7. Suppose $f_n(x) = \frac{x^2}{(1+x^2)^n}$ x real, n = 0, 1, 2, 3....
   Then the series $\sum_{n=0}^{\infty} \frac{x^2}{(1+x^2)^n}$:
   (A) converges point wise
   (B) converges uniformly
   (C) not (A)
   (D) not (B)

8. Suppose $f(x) = [x]$ for all $x \in [a, b]$ where a, b are real numbers with $a < b$. Then f is :

(A) Continuous on [a, b]

(B) Uniformly continuous on [a, b]

(C) Riemann integrable on [a, b]

(D) None

9. Suppose the set $\{v_1, v_2 ..... v_n\}$ spans a vector space V. Then dimension of V is :

(A) $= n$

(B) $> n$

(C) $< n$

(D) $\leq n$

10. Suppose T is a linear operator on R^2 defined by $T(x_1, x_2) = (x_1, 0)$. Let β be the standard basis and $\beta' = \{(1, 1), (2, 3)\}$ be any basis of R^2. Then the matrix of T relative to the pair β, β' is

(A) $\begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 1 \\ -1 & -3 \end{bmatrix}$

(D) None

11. (3, 0, 4), (–1, 0, 7) and (2, 9, 11) in R^3 are :

(A) orthogonal

(B) linearly independent

(C) neither (A) nor (B)

(D) both (A) and (B)

12. The values of λ, μ for which the following system of simultaneous equations has unique solution :

$x + y + z = 6;\ x + 2y + 3z = 0;\ x + 2y + \lambda z = \mu$

(A) $\lambda = 3, \mu \neq 3$

(B) $\lambda \neq 3$, any μ

(C) any $\lambda, \mu = 3$

(D) any λ, μ

13. If $f(t, x)$ is defined continuous on rectangle $R : \{(t, x) : |t - t_0| \leq a, |x - x_0| \leq b\}$ and is bounded by L on R and also satisfies Lipschitz condition on R then the initial value problem $x' = f(t, x)$, $x(t_0) = x_0$ has unique solution on the interval $I = |t - t_0| \leq h$ where h is :

(A) $\min\left(a, \frac{L}{b}\right)$

(B) $\min\left(b, \frac{L}{a}\right)$

(C) $\min\left(a, \frac{b}{L}\right)$

(D) $\min\left(b, \frac{a}{L}\right)$

14. The functions $x^4, |x| x^4$ are linearly independent on the interval :

(A) [0, 1]

(B) [–1, 0]

(C) [1, 2]

(D) [–1, 1]

15. The partial differential equation associated with $f(x + y + z), (x^2 + y^2 - z^2) >= 0$ is given by :

(A) $(y + z) p - (z + x) q = x - y$

(B) $(x + y) p + (y + z) q = z - x$

(C) $(x + y + z) p - (x - y - z) q = 0$

(D) $xp + yq = x + y + z$

16. The complete integral of partial differential equation $z = px + qy + p^2 + q^2$ by Charpit's method is given by :

(A) $z = ax + by + a^2$

(B) $z = ax + by + b^2$

(C) $z = ax + by + a^2 + b^2$

(D) $z = ax + by + ab$

17. Secant method to find a root of an equation :

(A) Converges always

(B) Diverges always

(C) May not converge some times

(D) None of these

18. For large n the Gauss elimination method requires :

(A) $\frac{n}{3}$ operations

(B) $\frac{n^2}{3}$ operations

(C) $\frac{n^3}{3}$ operations

(D) $\frac{n^4}{4}$ operations

19. If D is the differential operator and E is the shifting operator then hD is equal to :

(A) $\log E$

(B) $\log E^{-1}$

(C) $h \log E$

(D) $h \log E^{-1}$

20. Which of the following is Volterra integral equation ?

(A) $\phi(x) = \int_0^x k(x,t)\,\phi(t)\,dt$

(B) $\int_a^b k(x,t)\,\phi(t)\,dt + f(x) = \phi(x)$

(C) $\phi(x) = f(x) + \int_{-1}^{1} k(x,t)\,dt$

(D) None of these

21. The solution of I.E. $\phi(x) = x - \int_0^x (x-t)\,\phi(t)\,dt$, $\phi_0(x) = 0$:

(A) e^{x-t}

(B) e^x

(C) $\sin x$

(D) e^{-x}

22. The extremals of the functional $J[y(x)] = \int_1^3 (3x - y)\,y\,dy$ that satisfies the boundary conditions $y(1) = 1, y(3) = 9/2$ is :

(A) 1

(B) $\frac{9}{2}$

(C) $y = \frac{3}{2}x$

(D) No solution

3. On what curve can the functional $J[y(x)] = \int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx$ attain an extremum ?

(A) Parabola

(B) Hyperbola

(C) Ellipse

(D) None

24. The plane curve of fixed perimeter and maximum area is :

(A) Circle

(B) Ellipse

(C) Sphere

(D) Hyperbola

25. The number of generalized coordinates required to specify the configuration of rigid body moving in space are :

(A) 3

(B) 4

(C) 5

(D) 6

26. If $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.5$, $P(A\bar{B}) = 0.2$, $P(BC) = 0.3$, $P(\bar{A}\bar{B}\bar{C}) = 0.3$ and $P(AB|\bar{C}) = 0.1$ then $P(\bar{B}|\bar{C}) =$

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{1}{5}$

(D) $\frac{4}{5}$

27. Let x be a random variable with mean $\mu(<\infty)$ and variance $\sigma^2(<\infty)$ then for $\varepsilon > 0$:

(A) $P[|x-\mu| \geq \varepsilon] \leq \frac{\sigma^2}{\varepsilon^2}$

(B) $P[|x-\mu| \geq \varepsilon] \leq 1-\frac{\sigma^2}{\varepsilon}$

(C) $P[x-\mu \geq \varepsilon] \leq \frac{\sigma^2}{\varepsilon^2}$

(D) $P[x-\mu \geq \varepsilon] \leq 1-\frac{\sigma^2}{\varepsilon^2}$

28. The mode of F distribution is :

(A) $\frac{n_2}{n_2-2}$

(B) $\frac{(n-2)}{n_1}\cdot\frac{n_2}{(n_1+2)}$

(C) $\frac{n_1 n_2}{n_1+2}$

(D) $\frac{n_1}{n_1+n_2}$

29. If X is a random variable with Pdf $f(x) = e^{-x}$, $0 < x < \infty$ then the density of $y = x^3$ is :

(A) $\frac{1}{3y^{2/3}} e^{-y^{1/3}}$

(B) $\frac{1}{3} e^{-\frac{1}{3}y^{1/3}}$

(C) $\frac{1}{3} e^{-y^{1/3}}$

(D) $e^{-y^{1/3}}$

30. Stratified sampling belongs to the category of :

(A) Judgement sampling
(B) Subjective sampling
(C) Controlled sampling
(D) Non Random sampling

31. The formula for estimating a missing value in LSD with order k is :

(A) $R_i^1 + C_j^1 + T_k^1 - G^1 /(k-1)(k-2)$
(B) $k(R_i^1 + C_j^1 + T_k^1 - G^1)/(k-1)(k-2)$
(C) $k(R_i^1 + C_j^1 + T_k^1) - 2G^1/(k-1)(k-2)$
(D) $k(R_i^1 + C_j^1 + T_k^1) - kG^1/(k-1)(k-2)$

32. In a trivariate population, $r_{12} = 0.7, r_{13} = 0.6, r_{23} = 0.5$ then the multiple correlation $R_{1.23}$ is :

(A) 0.50
(B) 0.57
(C) 0.74
(D) 0.84

33. Mean source of an estimator $T_n(x)$ of $\psi(\theta)$ is minimum if:

(A) Bias and Var $(T_n(x))$ both are zero
(B) Bias is zero and Var$(T_n(x))$ is minimum
(C) Bias is minimum and Var$(T_n(x))$ is zero
(D) Bias and Variance both are minimum

34. The shape of Chi square distribution curve with d.f. 1 or 2 is :

(A) a parabola
(B) a hyperbola
(C) J shaped curve
(D) bell shaped curve

35. To test $H_0 : \mu < \mu_0$ against the alternative $H_1 : \mu > \mu_0$ H_0 is rejected at level α if :

(A) $\overline{x} \leq \mu_0 + \frac{s}{\sqrt{n}} z_\alpha$
(B) $\overline{x} \geq \mu_0 + \frac{s}{\sqrt{n}} z_\alpha$
(C) $\overline{x} \leq \mu_0 + \frac{s}{\sqrt{n}} t_{n-1,\alpha}$
(D) $\overline{x} \geq \mu_0 + \frac{s}{\sqrt{n}} t_{n-1,\alpha}$

36. Which one of the following tests is more efficient than sign test for testing for the Median of a continuous and symmetric distribution ?

(A) Wilconon signed Rank test
(B) Sukhatme's test
(C) Mood's test
(D) Wald Wolfowitz test

37. In a complete linkage clustering technique, the distance between two clusters can be measured as :

(A) $d_{(uv)w} = \text{Min}\{d_{uw}, d_{vw}\}$
(B) $d_{(uv)w} = \text{Max}\{d_{uw}, d_{vw}\}$
(C) $d_{(uv)w} = \frac{d_{vw} + d_{vw}}{2}$
(D) $d_{(uv)w} = \frac{|d_{uw} - d_{vw}|}{2}$

38. The number of primitive 100[th] roots of unity are :

(A) 40
(B) 60
(C) 100
(D) 99

39. G is a cyclic group of order 101. Then the automorphisms group of G has order :
(A) 101
(B) 2
(C) 100
(D) 1

40. Let G be a group, Z(G) its centre and $x \in Z(G)$. If C(x) is the conjugate class of x then order of C(x) is :
(A) 0(Z(G))
(B) 1
(C) 0
(D) |G|

41. A prime ideal in $(Z, +, \cdot)$ among the following is :
(A) $\langle 91 \rangle$
(B) $\langle 19 \rangle$
(C) $\langle 36 \rangle$
(D) $\langle 63 \rangle$

42. Consider the ring Z[i] of Gaussian integers. Then the number of units in Z[i] is :
(A) 1
(B) 2
(C) 3
(D) 4

43. The number of non-isomorphic non-abelian groups of order 8 is :
(A) 4
(B) 3
(C) 2
(D) 1

44. An irreducible polynomial among the following in Q[x] is :
(A) $2x^3 + 3x + 6$
(B) $x^4 - 3x^2 + 2$
(C) $x^3 - x^2 + x - 1$
(D) $x^3 + 1$

45. Let $K = Q(\sqrt{3}, i)$, $E = Q$ be fields. Then the degree of K over E is :
(A) 1
(B) 2
(C) 3
(D) 4

46. Consider the topology $J = \{\phi, X, \{1\}, \{1, 2\}\}$ of subsets of $X = \{1, 2, 3\}$. Then a closed set among the following is :
(A) $\{1\}$
(B) $\{1, 2\}$
(C) $\{2\}$
(D) $\{3\}$

47. $\lim_{z \to 2+\iota} \frac{z^3 - z - 10\iota}{z^2 - 4z + 5} =$
(A) $6 - 4\iota$
(B) $6 + 4\iota$
(C) $-6 - 4\iota$
(D) $-6 + 4\iota$

48. If $f(z) = u + \iota v$ is analytic in a domain with $u = -4xy - 2y$ then $f(z) =$
(A) $z^2 + \iota z$
(B) $2\iota (z^2 + z)$
(C) $2\iota (z^2 - z)$
(D) $z^2 + z$

49. Let C be the positively oriented circle $|z-2|=1$. Then $\int_C \frac{z}{2z-5}\,dz =$

(A) $5\pi i$

(B) $\frac{5\pi i}{4}$

(C) $\frac{5\pi i}{2}$

(D) $10\pi i$

50. Let $f(z) = z^2 e^{3z}$. Then $f^{(2016)}(0) =$

(A) 2015×3^{2014}

(B) $2015\times2014\times3^{2013}$

(C) 2014×3^{2013}

(D) $2016\times2015\times3^{2014}$